

More Mass. A Larger Horizon.

A Schwarzschild teaching model / radius, diameter and mass ratios

LEARNER 1 / PREDICT AND COMPARE

Name: _____ Date: _____

Allow 30–40 minutes with a calculator. This activity models isolated black holes with no spin or electric charge. The 10/20-solar-mass cases are hypothetical. The named cases use rounded mass comparisons, not new measurements.

1 / Predict the change

If mass doubles, does horizon diameter double, quadruple or stay fixed? Explain your prediction before calculating.



Common linear size scale: case 2 has twice the radius and diameter. Each circle represents a spherical horizon cross-section using areal radius r_s . It is not an image, a solid surface or a flat-space interior map.

2 / Complete the generated comparisons

Use $r_s \approx 2.953250076 \times m \text{ km}$, where m is mass in nominal solar units. Diameter $D = 2r_s$. Round sensibly; extra digits do not add measured precision.

Case / adopted mass	Radius / km	Diameter / km
1 / hypothetical, $m = 10$	_____	_____
2 / hypothetical, $m = 20$	_____	_____
3 / Sagittarius A*, $m = 4,000,000$	_____	_____
4 / M87*, $m = 6,500,000,000$	_____	_____

The named masses are rounded inputs. This simple Schwarzschild comparison does not fit their spin or an observed ring. Full tasks, answer discussion and sources: darksciencedocs.com/scale-lab/#horizon-teaching

A radius is half a diameter.

The mass determines the model size; zoom and display markers do not.

LEARNER 2 / CALCULATE AND INTERPRET

3 / Work the 10-solar-mass example

The lab uses the nominal solar mass parameter $\mu = 1.3271244 \times 10^{20} \text{ m}^3/\text{s}^2$, an exact conversion convention. With $c = 299,792,458 \text{ m/s}$ and adopted mass $m = 10$:

$$r_s = 2\mu m / c^2 \approx 29,532.500761 \text{ m} = 29.532501 \text{ km}$$

$$D = 2r_s \approx 59.065002 \text{ km} \approx 36.701290 \text{ miles}$$

μ already includes G . Do not multiply by G again. To convert km to international miles, divide by exactly 1.609344. In a spreadsheet, radius in km is:
 $=2*1.3271244E20*10/(299792458^2)/1000$

4 / Ratios, units and an inverse question

a. Compare case 2 with case 1 using D_2/D_1 . Compare Sagittarius A* with case 1 using only their adopted masses. Does the diameter ratio match the mass ratio?

b. For the Sagittarius A* comparison, $D \approx 23,626,000.609 \text{ km}$. Convert to AU using $1 \text{ AU} = 149,597,870.7 \text{ km}$. A different unit changes the numerical value; does it change the size?

c. A hypothetical Schwarzschild black hole has model $D \approx 118.130 \text{ km}$. Estimate m using $m = D / 5.906500152$, with D in km. Is this a mass measurement of a real object?

5 / Read the diagram, then limit the claim

In Scale Lab, use Black hole A = 10 and B = 20, then compare B = 4,000,000. The horizontal plot uses logarithmic position markers: their dot sizes do not show horizon sizes. Explain why a bright ring in a black-hole image is not a glowing horizon boundary.

Optional: horizon area is $A = 4\pi r_s^2$. If mass doubles, what happens to area? Explain why diameter and area have different scale factors. Do not infer an ordinary interior volume or a survival time.

Keep mass, size and image distinct.

Suggested ages 14+ through introductory college; scientific notation supported.

EDUCATOR 1 / FACILITATE AND ASSESS

Learning goals and timing

Learners distinguish radius from diameter, calculate model size in consistent units, use proportional reasoning and separate calculated horizons from observed emission/shadow structure. Prerequisites: multiplication, division and ratios. Give the numerical coefficient if exponent arithmetic is unfamiliar.

Prediction 5 min; cases 10; ratios and units 8; inverse question 5; image/limits discussion 7; optional area 5. Print pages 1–2 for learners. The paper cases work independently of the browser; no account or submitted student information is required.

Case	Radius / km	Diameter / km
10 nominal solar	29.532501	59.065002
20 nominal solar	59.065002	118.130003
Sagittarius A* comparison	11,813,000.304	23,626,000.609
M87* comparison	19,196,125,494.652	38,392,250,989.303

Displayed digits reproduce the recipe, not observational accuracy. Accept sensible rounding. A diameter is twice the radius in every row; μ is the mass parameter, so G must not be inserted again.

Suggested answers and reasoning

Case 2 / case 1 = 2. Sagittarius A* / case 1 = 400,000. M87* / Sagittarius A* = 1,625. The same ratios apply to radius, diameter and mass in this no-spin model. Sagittarius A* model diameter $\approx$ 0.157930059 AU (about 14,680,516.166 miles). Its physical model size is unchanged by the conversion.

The inverse example gives $m \approx 19.9999995$, so about 20 nominal solar masses. This is an assumed model diameter, not a measured real horizon. Doubling mass doubles r_s and D, but quadruples area. The horizon is a causal boundary; the bright ring involves emitting plasma and bent light, and the dark shadow is not the same boundary.

Assessment: require the factor of two between radius and diameter, consistent units, linear mass scaling and one valid limitation. A large calculator value is not evidence for a precise real mass. Logarithmic marker spacing and marker dot size must not be confused with a linear size drawing.

Check the arithmetic and assumptions.

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EDUCATOR 2 / REPRODUCE AND CREDIT

Offline verification

Extract the full kit. Python 3.9+ uses only its standard library:

```
python3 verify_analysis.py --data-dir .
```

```
Windows: py -3 verify_analysis.py --data-dir .
```

Compare the output JSON with worked-result.json. Four pinned conversions and four generated cases are checked for radius, diameter, miles, AU, ratios and inverse mass. The script changes no inputs and makes no network request.

The original Scale Lab model and its black-hole helper are included unchanged, with an ES-module declaration for optional Node inspection. Python does not need Node. This independent implementation checks a formula and stored transcriptions; it does not validate the model against observations.

Model boundaries

Assume an isolated, nonrotating, uncharged Schwarzschild black hole. The radius is an areal/circumferential coordinate: a sphere has area $4\pi r_s^2$. $D = 2r_s$ is the size convention used here, not a ruler distance through the interior. Real astrophysical black holes may spin. The lab does not calculate an accretion image, measured shadow, tidal survival or a new mass estimate.

Primary sources / checked October 25, 2026

Geometry / Andrew Hamilton, JILA: jila.colorado.edu/~ajsh/bh/schwp.html

IAU 2015 Resolution B3 / nominal mass parameter: arxiv.org/abs/1510.07674

BIPM / exact c: bipm.org/en/si-base-units/metre

IAU AU definition / Observatoire de Paris:

synte.obspm.fr/IAU_resolutions/Sumredef_au.html

NIST / international mile:

nist.gov/pml/us-surveyfoot/revised-unit-conversion-factors

EHT / Sagittarius A*:

eventhorizontelescope.org/blog/astronomers-reveal-first-image-black-hole-heart-our-galaxy

EHT / M87* Paper I: arxiv.org/abs/1906.11238

The lab adopts rounded 4 million and 6.5 billion nominal-solar comparisons. Exact conversion conventions do not remove the observational uncertainties in real mass estimates.

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