# Two Clocks. One Reunion.

Can a traveler return home having aged less than the people who stayed? Predict the reunion, compare two clock readings, and explain why arriving messages answer a different question.

Core: grades 9-12, about 50-60 minutes. Use a calculator and paper or the response text file. The printed cases work offline; the online lab is optional. Advanced high-school and introductory-college extensions are on page 5.

Original teaching materials by The Last Transmission: CC BY-NC 4.0. Copy and adapt for noncommercial educational use with attribution, a license link and a statement of changes. Full terms and exclusions: page 11.

Read or download the complete lesson
https://darksciencedocs.com/exploration-guides/time-dilation/

## Task 1 / Predict before calculating

Earth and the ship start together with clocks set to zero. The ship travels at 80% of light speed to a marker 6 light-years away in Earth's frame, turns around, and returns at the same speed. Predict which clock has accumulated more time at reunion. Explain what event makes the comparison unambiguous.

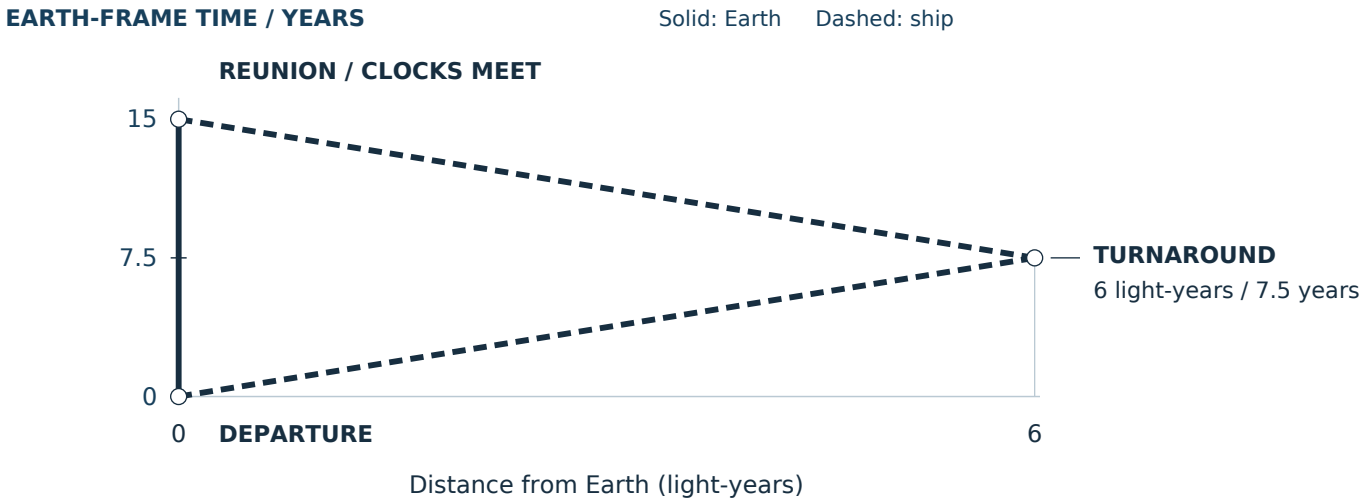



Diagram guide: vertical position is Earth-frame time; horizontal position is distance from Earth. Earth is the solid vertical path. The ship follows the dashed path. The drawing is a model, with the cruise and instantaneous-turnaround assumptions stated on page 2.

# Calculate the reunion

## Task 2 / Change one input; compare both clocks

Use L = 6 light-years for the one-way distance in Earth's frame. Let β = v/c be the fraction of light speed, $t_E$ the Earth elapsed time, and τ the crew elapsed time. Here, c = 1 light-year per year. A light-year measures distance; a Julian year is 365.25 days.

**EARTH ELAPSED TIME**

$$t_E = \frac{2L}{\beta}$$

**CREW ELAPSED TIME**

$$\tau = t_E\sqrt{1-\beta^2}$$

For the 80% case, calculate Earth time as 2 × 6 ÷ 0.8 years. Then find the square-root factor, crew time, and the difference between the reunion clocks. Keep years in every time answer.

### Generated return missions; complete the two blank columns

| Speed β | Earth years $t_E$ | Crew years τ | Difference, years |
|---|---|---|---|
| **0.50** | 24.0000 | Calculate | Calculate |
| **0.80** | 15.0000 | Calculate | Calculate |
| **0.99** | 12.1212 | Calculate | Calculate |

a. Rank the missions by crew aging. b. If distance stays fixed, how do Earth time and crew time change as speed increases? c. Would a longer-distance voyage at a lower speed necessarily age the crew less? State which inputs must be controlled.

## Optional online check

Choose Classroom settings near Worksheet & educator notes, or set the workbench to 80% c and 6 light-years. Select Compare at reunion. For messages, use the same values in Plan your flight, then Launch & watch Earth messages. These are separate experiments; check both settings. The 3D deck is optional.

## The model boundary

The model uses two constant-speed cruise legs in an inertial Earth frame. Instantaneous turnaround is an idealized limit. Gravity, propulsion, finite acceleration, shielding and energy budgets are omitted. Each traveler's local clock ticks normally; the ship changes inertial frames at turnaround.

© 2026 DarkScienceDocs.com / CC BY-NC 4.0 / original teaching materials

# Messages are not clock readings

## Task 3 / Distinguish transit from the wait between receipts

For the 80% mission, Earth sends a numbered message every 0.5 Earth year (6 months), beginning with #1 at 0.5 year. Each carries its send timestamp and travels at c in vacuum. Other missions may use a different interval; read the cadence label before comparing.

**Generated events; all times are elapsed years since the shared departure**

| Message | Earth send time, s | Earth-frame reception, $t_r$ | Crew clock at reception |
|---|---|---|---|
| **#1** | 0.5000 | 2.5000 | 1.5000 |
| **#2** | 1.0000 | 5.0000 | 3.0000 |
| **#3** | 1.5000 | 7.5000 | 4.5000 |
| **#4** | 2.0000 | 7.7778 | 4.6667 |
| **#29** | 14.5000 | 14.7222 | 8.8333 |
| **#30** | 15.0000 | 15.0000 | 9.0000 |

Earth-frame reception time is the coordinate time of the receipt event away from Earth, assigned in Earth's frame. It is not the older Earth timestamp carried in the arriving message. #3 arrives exactly at turnaround; #30 is sent and received at the reunion point.

a. For messages #1, #2, #3, #4, #29 and #30, subtract the Earth send time from the Earth-frame reception time. This gives each message's transit duration. Does it decrease for successive return-leg messages?

b. Subtract consecutive crew receipt readings for #1→#2, #2→#3, #3→#4 and #29→#30. Convert the waits to months using 12 months per year. Why can the return-leg wait stay constant while individual transit durations decrease?

**WAIT MEASURED ON THE CREW CLOCK**

#1 to #2: 18 months



#3 to #4: 2 months



Same time scale. Every signal travels at c in vacuum.

c. Can an Earth send-clock reading be subtracted from a crew receive-clock reading to obtain transit time? Explain the time convention in each calculation. Rows #5-#28 are omitted; #4 and #29 are not consecutive.

© 2026 DarkScienceDocs.com / CC BY-NC 4.0 / original teaching materials

# Compare gravity; defend a claim

## Task 4 / Two contributions to a GPS clock

In Gravity & clocks, choose GPS satellite. Clock A is on the ground; Clock B orbits Earth. Compare the signed numerical contributions. The actual hand drift is tiny; any magnified animation is explicitly illustrative. Do not infer microseconds by counting animation frames.

**Rounded reference comparison from NIST; not the exact output of every orbit model**

| Contribution to satellite B relative to ground A | Approximate offset per day |
|---|---|
| **Weaker gravity at the satellite** | +45 microseconds |
| **Satellite’s orbital motion** | -7 microseconds |
| **Combined offset** | Calculate |

a. Add the two contributions. Does B gain or lose time relative to A without compensation? b. Explain why “every higher clock runs slower” is not a valid rule. c. Name one additional effect omitted from this rounded classroom comparison.

## Task 5 / Claim, evidence, reasoning and boundary

A classmate says: “The ship gets messages faster coming home, so the crew must have aged more than Earth.” Evaluate this claim using the reunion clocks and two receipt events. Include the relevant time units, one idealization and a measurement that would be needed to test a real journey.

## Exit check

1. Does a photon carry a working clock or have a rest frame? 2. Does zero proper time along a lightlike path mean a signal reaches the ship instantly in Earth’s frame? 3. At reunion, which readings give each traveler’s accumulated aging?

Optional: compare rest and motion in the uncalibrated flight-deck sky.

© 2026 DarkScienceDocs.com / CC BY-NC 4.0 / original teaching materials

# Design a mission; derive a receipt

## Task 6 / Algebra and introductory-college extension

Design an ideal cruise mission that returns after 100 Earth years while accumulating 10 crew years. Rearrange the crew-time equation on page 2 to make β the subject. Show the algebra, substitute the target ratio, then use L = β $t_E$ / 2 to find the one-way distance in light-years.

---

The online mission controls have finite slider precision. Select the nearest supported speed and distance, recalculate the actual clocks, and judge the stated challenge tolerances. Mathematical target values do not make the acceleration or energy requirements achievable.

## Optional derivation / Where does a message meet the ship?

For the 80% mission, let t be Earth-frame years and x be light-years from Earth. Use $x_{ship}$ = 0.8t outbound, and $x_{ship}$ = 12 - 0.8t on return. A signal sent at Earth time s follows $x_{light}$ = t - s. Set the two positions equal to derive each reception formula. State the send-time boundary that selects the correct cruise leg.

Check your formulas with messages #1, #3 and #4. Then derive the constant crew wait for consecutive messages on each cruise leg. A local inertial-frame comparison is not one global rest frame for the entire returning ship.

---

## Optional gravity calculation / Two static hovering clocks

Clock B is held at four Schwarzschild radii ($r_B$ = 4 $R_S$); clock A is held at twenty ($r_A$ = 20 $R_S$). $R_S$ is the black hole's horizon radius. Both clocks hover at fixed positions around a nonrotating black hole; neither is orbiting. Compare B with the finite-radius clock A, not with a clock at infinity.

**NEAR CLOCK B / FAR CLOCK A**

$$\frac{\text{rate}_B}{\text{rate}_A} = \sqrt{\frac{1-\frac{1}{4}}{1-\frac{1}{20}}}$$

How many minutes elapse on B while A advances one hour? Compare the percentage effect with the GPS microsecond-scale effect. The lab's black-hole Comparison interval is in days on the farther clock; normalize to one hour for this calculation. Strong-field hovering and circular GPS orbits use different models.

© 2026 DarkScienceDocs.com / CC BY-NC 4.0 / original teaching materials

# Facilitate the investigation

## Levels and routes

Grades 9-12 / ages about 14-18: pages 1-4 form the core. Prerequisites are ratios, signed arithmetic, square roots and reading a table. Introductory college or advanced high school: add page 5’s formula rearrangement and piecewise intersection derivation. No calculus is required.

Grades 6-8 / ages about 11-14: teacher-led conceptual enrichment. Supply the calculated crew values, compare and rank the cases, and discuss receipt cadence without deriving the Lorentz factor. Do not report this as completion of a separate middle-school performance expectation. Homeschool and independent learners can use the same level choices.

## Suggested 50-60 minute core

Prediction and definitions: 8 minutes. Reunion calculations: 12. Message events: 15. GPS comparison: 8. Claim, discussion and exit check: 12. Optional extensions add about 20-35 minutes. These are planning estimates; the lesson has not been classroom-timed.

Print pages 1-4 for the core and page 5 only when needed; retain pages 6-11 for teaching and reference. Use a calculator, paper or the plain-text response file. Shared-device, teacher-projected and fully offline routes are supported. Student answers are not collected by this resource.

## Answers / Tasks 1 and 2

Reunion is one shared event, so the co-located clocks can be compared directly. The crew has accumulated less proper time. For $\beta = 0.8$, the square-root factor is 0.6, $\gamma$ is 5/3, Earth time is 15 years and crew time is 9 years. The difference is 6 years. At turnaround the crew clock reads 4.5 years and Earth-frame time is 7.5 years.

**Reference mission arithmetic; accept sensible rounding**

| β | Earth years | Crew years | Difference, years |
|---|---|---|---|
| **0.50** | 24.0000 | 20.7846 | 3.2154 |
| **0.80** | 15.0000 | 9.0000 | 6.0000 |
| **0.99** | 12.1212 | 1.7099 | 10.4113 |

At fixed distance both times decrease as speed increases, while the crew-to-Earth ratio also decreases. Speed alone cannot rank aging for journeys with different distances. The round trip is asymmetric because the ship changes inertial frames; acceleration is not a magical slowing substance. Access and response options are on page 11.

© 2026 DarkScienceDocs.com / CC BY-NC 4.0 / original teaching materials

# Separate the two durations

## Answers / Tasks 3-5

Earth-frame transit durations for #1, #2, #3, #4, #29 and #30 are 2.0000, 4.0000, 6.0000, 5.7778, 0.2222 and 0 years. The late zero is the reunion endpoint at zero separation. Do not generalize it to distant messages.

The crew waits #1→#2 and #2→#3 are 1.5 years = 18 months each. #3→#4 and #29→#30 each take about 1/6 year = 2 months. For uniform cruise speed and a regular emitter, receipt spacing is constant on a cruise leg. Individual return-leg photon paths still shorten, so transit durations decrease. All photons travel at c.

A crew wait subtracts two events on the same crew clock. Earth-frame transit subtracts Earth-coordinate send and receipt times. Subtracting an Earth send reading from a crew receive reading mixes clock conventions and is not either of these durations.

The GPS reference sum is +38 microseconds per day: the satellite gains time without compensation. Weaker gravity makes the satellite clock run faster; orbital motion opposes this. Exact orbit, Earth rotation, nonspherical gravity and hardware corrections need a fuller model. NIST’s rounded example is separate from the generated voyage cases.

A strong Task 5 response rejects the classmate’s inference, uses the 15-year/9-year reunion and a receipt comparison, states an idealization and proposes a clock or signal observation. No photon rest frame exists. A null path’s zero proper time does not eliminate Earth-frame propagation time.

## Suggested 12-point rubric

### Score each criterion 0-2; partial evidence earns 1

| Criterion | Evidence for 2 points |
|---|---|
| **Model and reunion** | Identifies shared reunion, cruise legs and a model limit. |
| **Clock calculation** | Finds Earth 15 y, crew 9 y; keeps units consistent. |
| **Transit duration** | Subtracts Earth-frame send time from reception time; return values decrease. |
| **Receipt cadence** | Finds 18 then 2 crew months; does not change c. |
| **GPS contributions** | Combines signs to +38 microseconds/day. |
| **Explanation and limits** | Uses numerical evidence; separates clock aging from received light. |

The rubric records evidence within this activity, not standards mastery or external certification. Assess optional algebra separately, only when learners rearrange the formula rather than substitute into it.

© 2026 DarkScienceDocs.com / CC BY-NC 4.0 / original teaching materials

# Standards / scientific practice

Publisher-authored mapping for the grades 9-12 core and optional algebra extension. The lesson supports selected NGSS science practices and crosscutting concepts. It does not claim an entire three-dimensional NGSS performance expectation, external approval or college accreditation.

### Selected framework components and assessable evidence

| Target | Task and evidence |
|---|---|
| **NGSS SEP 2 / Developing and using models** | Tasks 1, 2, 5: worldline prediction, parameter comparison and an explicit model boundary. |
| **NGSS SEP 4 / Analyzing and interpreting data** | Task 3: interprets generated send/receipt events and compares both durations. |
| **NGSS SEP 5 / Mathematics and computational thinking** | Tasks 2-4: clock factors, time differences, months and signed GPS contributions. |
| **NGSS SEP 6 / Constructing explanations** | Task 5: a numerical claim connected to a principle, assumption and needed observation. |
| **NGSS CCC 3 / Scale, proportion and quantity** | Tasks 2-4: light-years versus years, clock ratios and microseconds versus years. |
| **NGSS CCC 4 / Systems and system models** | Tasks 1, 5: defines Earth, ship, signals, inputs and omitted physics. |

Assess these components through the scientific work, not through memorizing the names of practices. A generated model result should be distinguished from an empirical measurement. The separate NIST clock reference provides a real-world comparison.

Official NGSS practices and crosscutting-concept appendices
https://www.nextgenscience.org/resources/ngss-appendices

© 2026 DarkScienceDocs.com / CC BY-NC 4.0 / original teaching materials

# Standards / mathematics and levels

These are selected mathematics connections tied to completed tasks. A numerical answer without units or reasoning does not, by itself, establish mastery of a standard.

### Mathematics targets and assessable evidence

| Target | Student work |
|---|---|
| **CCSS HSN.Q.A.1 / Quantities** | Tasks 2-4: consistent units and time origins in multistep calculations. |
| **CCSS HSA.CED.A.4 / Rearranging formulas** | Task 6 only: isolates β from the crew-time equation, then checks the target. |

## Choose a route for your learners

### Planning choices; times are estimates

| Level | Suggested route | Mathematical support |
|---|---|---|
| **Grades 6-8 enrichment** | Teacher-led predictions, supplied clock values and message comparisons; about 30-40 minutes. | Ratios and reading tables. No separate middle-school PE claim. |
| **Grades 9-12 core** | Learner pages 1-4; about 50-60 minutes. | Ratios, signed arithmetic, square roots and consistent units. |
| **Advanced high school / introductory college** | Core plus page 5; add about 20-35 minutes. | Rearrange a formula; derive piecewise signal intersections. No calculus needed. |

Homeschool and independent learners may use any appropriate route. The introductory-college label describes intended use, not accreditation. Claim the formula-rearrangement connection only if Task 6 is completed; substituting into a supplied expression is not that task.

Official Common Core quantities targets
https://www.thecorestandards.org/Math/Content/HSN/Q/

Official Common Core creating-equations targets
https://www.thecorestandards.org/Math/Content/HSA/CED/

© 2026 DarkScienceDocs.com / CC BY-NC 4.0 / original teaching materials

# Check the model independently

## Extension answers / Task 6 and static gravity

**TARGET SPEED / 100 EARTH YEARS, 10 CREW YEARS**

$$\beta = \sqrt{1 - \left(\frac{\tau}{t_E}\right)^2} = \sqrt{0.99} \approx 0.9949874371$$

The exact one-way distance is L = 49.74937186 light-years. Slider rounding can change the result: report the tested settings and clocks. These ideal target values do not establish mission feasibility.

Outbound: $t_r$ = 5s, for Earth send times through 1.5 years.
Returning: $t_r$ = (12 + s)/1.8. Crew receipt time is 0.6 $t_r$.
The send-time boundary gives reception at turnaround. For static clocks, the rate ratio is 0.8885233; B advances 53.3114 minutes per hour on A.

## Offline verification

Extract the teaching kit and run python3 verify_analysis.py (Windows: py -3 verify_analysis.py). Python 3.9+ with its standard library is sufficient. The checker tests signal intersections, proper-time factors and CSV rounding; it makes no network request and does not alter the supplied data.

The cases are generated ideal-model data, not telemetry. NIST's separate GPS comparison connects the lesson to measured technology. The kit includes the complete source links.

## Scientific sources

Einstein Online: the travelling twins
https://www.einstein-online.info/en/spotlight/twins/

Einstein Online: light clocks and time dilation
https://www.einstein-online.info/en/spotlight/light-clocks-time-dilation/

NIST: atomic-clock tests and the GPS comparison
https://www.nist.gov/atomic-clocks/a-powerful-tool-for-science/putting-einstein-test

MIT 8.962: Schwarzschild clocks, section 6.2.2
https://web.mit.edu/8.962/www/lecnotes/8_962TA-lec-all.pdf

## Standards sources

NGSS high-school practice elements
https://www.nextgenscience.org/sites/default/files/HS%20Practices%20Combined.pdf

NGSS appendices and crosscutting concepts
https://www.nextgenscience.org/resources/ngss-appendices

Common Core targets are linked on page 9. The issuing organizations own their standards; descriptions here are paraphrased. Sources reviewed November 7, 2026.

© 2026 DarkScienceDocs.com / CC BY-NC 4.0 / original teaching materials

# Access, license and attribution

## Accessible formats and response choices

Use the semantic HTML lesson for reflow, selectable text, headings, table headers and diagram descriptions. The standalone HTML in the kit works offline. Use learner-responses.txt for local typed responses, or write on paper or dictate. No account, purchase or online answer submission is required.

The PDF uses tagged headings, tables with scoped headers, a logical reading sequence, descriptive links, heading bookmarks, embedded fonts, document language and text alternatives for figures and formulas. Running furniture and writing rules are marked as artifacts. Meaning is carried by labels and line styles as well as color.

Print on US Letter, 8.5 × 11 inches. For A4, choose Fit. Text and diagrams remain usable in grayscale. For larger type or high zoom, use the HTML edition or reflow where supported. The full online lab uses JavaScript; the core printed lesson does not need JavaScript, WebGL, animation or sound.

## Review status

Reviewed November 7, 2026. PAC 2026 PDF/UA, WCAG and Quality checks report no errors or warnings. NVDA sound/reading check completed.

## Creative Commons educational license

© 2026 DarkScienceDocs.com. Original lesson text, generated tables, response template and instructional diagrams by The Last Transmission are licensed under Creative Commons Attribution-NonCommercial 4.0 International. You may copy, distribute and adapt them for noncommercial purposes with appropriate credit, a license link and a statement of changes.

Read the CC BY-NC 4.0 license
https://creativecommons.org/licenses/by-nc/4.0/

This grant does not license the website software, branding, documentary footage, third-party standards or linked source material. Their separate terms continue to apply. Reused editorial artwork carries its own credit and is not part of the PDF’s original mathematical diagrams.

## Suggested attribution

The Last Transmission, “Two Clocks. One Reunion.” Time Dilation teaching lesson, DarkScienceDocs.com, CC BY-NC 4.0. Add the resource link and identify adaptations.

Permanent lesson and current downloads
https://darksciencedocs.com/exploration-guides/time-dilation/